Math 14 **Sample Gateway Exam 1** **100 Points**

All numbered problems are worth ten points except problem 9, which is worth 20 points. You must get at least 90 points to pass.

1. $\displaystyle\int_0^4 x\sqrt{9+x^2}\,dx =$

2. $\displaystyle\int \frac{x^3}{\sqrt{4-x^2}}\,dx =$ Use $x = 2\sin u$.

3. $\displaystyle\int x^3 \ln x\,dx =$

4. $\displaystyle\int_2^\infty xe^{-x^2}\,dx =$

5. a) $\displaystyle\lim_{x\to 0}\frac{\sin x}{x+\sin x} =$ b) $\displaystyle\lim_{x\to 0}\frac{\cos x}{x+\cos x} =$

6. $\displaystyle\frac{e}{\pi}+\frac{e^2}{\pi^2}+\frac{e^3}{\pi^3}+\cdots+\frac{e^n}{\pi^n}+\cdots =$

7. Find the interval of convergence of $\displaystyle\sum_{k=0}^{\infty}\frac{k(x+5)^k}{2^k}$ except for the endpoints.

8. Find the power series expansion about $x = 0$ of the function $f(x) = 3e^{2x}$.

9. For each of the following series state whether it

   A) converges absolutely, B) converges conditionally, or C) diverges.

   You do not need to justify your answers.

   a) $\displaystyle\sum_{k=0}^{\infty}\frac{(-2)^k}{k!}$ b) $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^k k^2}{k^3-5k+3}$ c) $\displaystyle\sum_{k=1}^{\infty}\frac{k}{1+3k}$ d) $\displaystyle\sum_{k=1}^{\infty}\frac{\sin k}{k^2}$